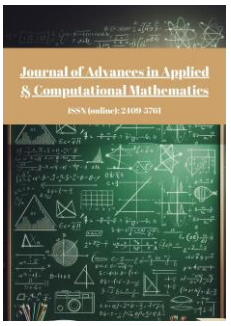
Published by Avanti Publishers

Journal of Advances in Applied & Computational Mathematics

ISSN (online): 2409-5761



A Systems-Theoretic PMOO Framework for Decision-Making under Uncertainty: A Comprehensive Multi-Scenario Approach

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ARTICLE INFO

Article Type: Research Article

Academic Editor: Youssef Raffoul

Keywords:
Robustness
Systems theory
PMOO methodology
Multi-scenario analysis
Decision-making under uncertainty

Timeline:
Received: October 08, 2026
Accepted: November 20, 2026
Published: November 27, 2026

Citation: Zheng M, Yu J. A systems-theoretic PMOO framework for decision-making under uncertainty: A comprehensive multi-scenario approach. J Adv Appl Computat Math. 2026; 13(2): 213-226.

DOI: https://doi.org/10.15377/2409-5761.2026.13.13

ABSTRACT

The traditional decision-making methods under uncertain scenarios include optimism criterion, pessimism criterion, compromised criterion, equal likelihood criterion, and minimax regret criterion. However, each of these criteria could only supply solution either extreme option or compromised choice instead of considering all scenes together comprehensively, which are not quite practical. Here in this article a virtual system is constructed in view of "higher dimensional space", which includes all the uncertain scenarios as independent events / attributes first. It enables us to propose a new quantitative approach for decision-making under uncertain scenarios in viewpoint of systems theory based on PMOO rationally. The new approach is an overall approach comprehensively considering all scenarios in coordinate way and relatively stable (with robustness) with respect to the variation of scenario, which is compatible with all the uncertain scenarios comprehensively instead of excluding any one of the scenarios. In the approach, the quantitative decision - making with risk and discussion on utility estimation are given as well.

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1. Introduction

Generally speaking, according to the content of decision quantification, decision-making can be divided into deterministic decision-making and decision-making under uncertainty, and risky decision [1-3].

The word "decision-making" is in fact not unfamiliar to people. It is an activity that people will encounter in many fields, such as daily life, economy, science and technology, and even politics and national defense. It means to find the "best" or "most satisfactory" scheme from multiple schemes. Decision-making is to take choice and decision, which involves a series of activities such as thinking, calculation, and comparison, etc. Reasonable decision-making can achieve the effect of "getting efficient achievement with effective effort", while improper decision-making may cause greater losses. This kind of loss is not necessarily economic, nor can it be measured by economy, and some of them even have aftereffects.

It is quite often for people to make their decision (choice) from many alternatives under uncertain conditions. Besides, with the development of society, production, science and technology, the decision-making problems become more complicated or complex. It seems difficult for people to make appropriate decision only based on their perceptual knowledge and personal experiences. As a result, some scientific decision-making ideas and methods have emerged. Decision analysis is a set of concepts and systematic methods to help people to analyze the structure of decision - making problems rationally and choose proper evaluation criteria and decision - making methods.

When facing to decision-making problem, its consequence does not only depend on the decision-maker's choice strongly, but also significantly related to external factors. The external factors are either unknown or beyond the decision maker's control when he makes a decision. This external factor is called natural state in general. The choice always has some preferences, such as "seeking good fortune and avoiding evil", "seeking advantages and avoiding disadvantages", "avoiding heavy and taking the light" and so on. Therefore, it is really a kind of "activity" with strong subjectivity or preference, which has to conduct the combination of subjectivity with objectivity properly. Moreover, in a certain sense, it is also an optimization activity or process. However, different people's understanding or cognition of "auspiciousness" and "benefit" will have a great gap. The meaning of "benefits" is dual folds, it has long-term interests and short-term interests, as well as the overall (mass, public, overall situation and collective) interests and local (short-term, local and personal) interests. "Auspicious" also has the difference between overall peaceful coexistence, equality and harmony, and exclusive and exclusive prosperity. Therefore, it is not surprising that because of this different understanding of "seeking good fortune and avoiding evil" and "seeking advantages and avoiding harm", different choices or decision-making orientations appear, and diverse consequences present.

At present, decision making under uncertainty is making decisions by subjective intention when the probability of the outcome that will happen is uncertain or ignorant.

Risk-based decision-making is a decision-making when the natural environment is not completely determined, but the probability of its occurrence can be pushed or predicted. Objectively speaking, the meaning of "risk" is also in two-way. For different people, the degree of perception of the same thing is different, or even opposite. For example, for the same commodity (item), when the price drops, the merchant feels that the risk is bigger or very big, while the buyer feels that it is good. People who don't buy this product have no feeling.

Up to now quantitative decision-making methods under uncertain scenarios provide optimism criterion, pessimism criterion, compromised criterion, equal likelihood criterion, and minimax regret criterion in general decision-making analysis, each provides an independent solution rather than considering all the scenes together comprehensively. Of course, these selections are problematic for practical application due to the actual situation being unclear. Therefore, an appropriate selection is in need for these unidentified scenarios.

In fact, even if the future scenario is unknown or unidentified at present, we have to make a decision, therefore we can choose a proper scheme from the limited schemes, which has the characteristics of comprehensively considering various scenarios together in viewpoint of systems theory rather than any extreme solution, that is, a scheme that could consider the effects of each scenario comprehensively and take effect of all scenarios into account, or a robust solution that is relatively stable despite of actual appearance of any scenario.

Through serious analysis, it finds that PMOO may provide a new approach to solve this problem with each scenario as an independent "event" in probability theory [4]. In the evaluation of probabilistic multi – objective optimization (PMOO), it focuses to find out the partial preferable probability of the utility value of each objective (or benefit) and the total preferable probability of each scheme, and then select the one which is with the highest total preferable probability.

In order to deal with the matter, here in this article a virtual system is constructed in view of "higher dimensional space", and then a new approach is proposed to provide a rationally quantitative decision-making under uncertain scenarios in viewpoint of systems theory, which is an overall approach based on PMOO comprehensively considering all scenarios together in coordinate way [4].

Simple examples are employed to demonstrate the approach in detail.

2. An Outline of the Quantitative Decision-making Methods Under Uncertain Scenarios

Till now the traditional quantitative decision-making methods under uncertain scenarios give optimism criterion, pessimism criterion, compromised criterion, equal likelihood criterion, and minimax regret criterion for decision-making analysis in general, each of which provides its solution meeting its specific demand rather than comprehensively considering all scenes together. This case indicates that the solution determined by each of the above mentioned criterion does not need to be compatible with other scenarios, i.e., it is unstable or non-robust with respect to the variation of scenarios. In following contents of this section it is outlined first from a simple example for our comparison shortly.

2.1. Optimism Criterion

The optimism criterion or maximax criterion conducts its option like this [1-3]: the decision-maker has an optimistic and adventurous attitude. He will never give up any chance to get the best result, and strive to choose the scheme with the attitude "from good to great" in his decision-making. From the decision table, the decision-maker selects the maximums from the results of all states in each scheme, then lists them, and compares all these maximum values, and finally further selects the greatest one from all these maximum values as his choice, which corresponds to his final optional scheme.

An example is shown in Table **1**, there are 5 schemes (K_1, K_2,..., K_5) and four natural states (τ_1, τ_2,..., τ_4). As a result, the maximum profit value is $\max\max_{j=1;i=1}^{4;5}(K_i,\tau_j)=\max\{7,9,7,8,5\}=9$, which corresponds to the scheme K_2.

Table 1: A decision profit table for optimism criterion.

Profit / Scheme	*State*				$\max_{j=1}^{4}(K_i,\tau_j)$
	τ_1	τ_2	τ_3	τ_4	
K_1	4	5	6	7	7
K_2	2	4	6	9	9*
K_3	5	7	3	5	7
K_4	3	5	6	8	8
K_5	3	5	5	5	5

2.2. Pessimism Criterion

The pessimism criterion or maximin criterion conducts its option as follows [1]: the decision-maker has a pessimistic and conservative attitude. He selects the minimums from the results of all natural states of the table in

each scheme, lists them, then compares all these minimums, and further selects the maximum from all these minimums, and determines the corresponding scheme as its choice finally. Again, take the same example, i.e., it involves 5 schemes (K_1, K_2,..., K_5) and four natural states (τ_1, τ_2,..., τ_4). The choice in pessimism criterion is shown in Table **2**, As a result, the maximum profit value is $\max\min_{j=1;i=1}^{4;5}(K_i,\tau_j)=\max\{4,2,3,3,3\}=4$, which corresponds to the scheme K_1.

Table 2: A decision profit table for pessimism criterion.

Profit / Scheme	State				$\min_{j=1}^{4}(K_i,\tau_j)$
	τ_1	τ_2	τ_3	τ_4	
K_1	4	5	6	7	4*
K_2	2	4	6	9	2
K_3	5	7	3	5	3
K_4	3	5	6	8	3
K_5	3	5	5	5	3

2.3. Compromised Criterion

In compromised criterion (Hurwicz criterion), it considers neither optimistic nor pessimistic about the development and evolution of things, but seek a "compromised balance" from the above two cases, and use a compromise coefficient α to reconcile, $CV_i=\alpha\, max_{j=1}^{m}a_{ij}+(1-\alpha)\, min_{j=1}^{m}a_{ij}$, $\alpha\in[0, 1]$, to get CV_i, and finally the largest one of CV_i and the corresponding scheme from all schemes are chosen, in which a_{ij} is the matrix element of the profit table, and m is the natural state number of each scheme. Table **3** shows the selection result of subjective $\alpha = 0.8$ for the same example. As a consequence, $\max CV_{i=1}^{5}(K_i,\tau_j)=\max\{6.4,7.6,6.2,7,4.6\}=7.6$, which corresponds to scheme K_2.

Table 3: A decision profit table for compromise criterion.

Profit / Scheme	State				CV_i
	τ_1	τ_2	τ_3	τ_4	
K_1	4	5	6	7	6.4
K_2	2	4	6	9	7.6*
K_3	5	7	3	5	6.2
K_4	3	5	6	8	7
K_5	3	5	5	5	4.6

4) Equal Likelihood Criterion

Equal likelihood criterion or Laplace criterion holds that when the decision-maker cannot determine the probability of each natural state in advance, he can assume that the probability of each natural state is equal, that is, $1/m$, and m is the number of natural states, and then make decisions according to the maximum expectation criterion (i.e., weighted average value). The result for the same example is given in Table **4**, and the scheme K_1 is finally selected. It is worth to mention that although the expected values $E(K_i)$ of the schemes K_1 and K_4 are the same, says 5.5, it is obvious that the scheme K_1 is more stable to the natural states than the scheme K_4, so the scheme K_1 is selected. The maximum fluctuation $D(K_i)$ is used to reflect the stability of a scheme, as is listed in the last column of Table **4**.

Table 4: A decision profit table for equal likelihood criterion.

Profit / Scheme	State				$E(K_i)$	$D(K_i)$
	τ_1, $P_{\tau 1}$=1/4	τ_2, $P_{\tau 2}$=1/4	τ_3, $P_{\tau 3}$=1/4	τ_4, $P_{\tau 4}$=1/4		
K_1	4	5	6	7	5.5*	1.5
K_2	2	4	6	9	5.25	3.75
K_3	5	7	3	5	5	2
K_4	3	5	6	8	5.5	2.5
K_5	3	5	5	5	4.5	1.5

2.5. Minimax Regret Criterion

In minimax regret criterion or Savage criterion, the policymakers believe that after making a decision, if it is not in line with the actual situation, there will inevitably be a feeling of regret. The characteristic of this method is to take the maximum profit value of each natural state (the minimum loss matrix) as the ideal goal of that natural state, while the difference between the profit value and the ideal value of other schemes in the same natural state is its regret value. So we can get a corresponding regret matrix from the profit matrix. Finally, compare the maximum regret value of each natural state, and then find the minimum one as the option. Table **5** shows the case for the same example by setting the regret matrix in the lower part. As a result, $min\{2,3,4,2,4\} = 2$, thus K_1 or K_4 is optioned.

Table 5: A decision profit table for minimax regret criterion.

Profit / Scheme	τ_1	τ_2	τ_3	τ_4
	State			
K_1	4	5	6	7
K_2	2	4	6	9
K_3	5	7	3	5
K_4	3	5	6	8
K_5	3	5	5	5
	Regret Matrix			
K_1	1	2	0	2
K_2	3	3	0	0
K_3	0	0	3	4
K_4	2	2	0	1
K_5	2	2	1	4

2.6. Analysis

About results indicate that each criterion sticks to its concerns rather than all scenes, and specific solution may be got correspondingly, but the future is unidentified, and the specific solution is not compatible with other scenarios in general, this is the resource of so called “regret” in the minimax regret criterion if the his solution is not in line with the actual situation. Facing to unidentified scenario, it might be better to make a proper decision which is compatible with all scenarios.

3. Quantitative Decision-making Under Uncertain Scenarios with PMOO

In section 2, the results from the traditional (previous) criteria are presented, which provide either extreme choices or compromised option instead of comprehensively considering all scenes together. Of course, each of these selections could be applicable to specific case. However, since the actual case in future is unidentified, it is difficult for one to make a precise decision for unidentified scenario, and each specific solution of the traditional methods is not compatible with other scenarios in general, therefore, it is the resource of so called “regret”.

In such case, an appropriate way is to find a relatively satisfactory solution that is compatibility with all scenarios beneficially and robustly. This section aims to deal with this problem.

3.1. Idea of Appropriate Solution for Decision - Making with PMOO Under Unidentified Scenarios

Since the actual scenario in the future is unknown or unidentified, we can choose a proper scheme from the limited alternative candidates, which is compatibility with all scenarios beneficially and robustly. Therefore, the basic requirement for the selected scheme is to be compatible with all the uncertain scenarios comprehensively rather than excluding any one of the scenarios. The compatibility of the scheme means robustness and suitability despite of the variations of scenario. The way to find such a precious solution is to comprehensively consider all the scenarios together rather than only beneficial to any extreme scene alone, that is, a scheme that could systemically take the needs of all scenarios into account.

In order to conduct the comprehensive consideration of all scenarios, each natural state must be accommodated equivalently at the same time in virtual system of a “higher dimensional space”. Thereafter, we could use probabilistic multi – objective optimization (PMOO) approach to handle the matter. The details of the operation is as follows,

a) Construct a virtual system in view of “higher dimensional space”, which includes multiple objectives, each objective corresponds to one natural state (scenario) τ_i, $i = 1, 2,\ldots, n$, i.e., as pseudo - independent events / attributes;

b) Take each scenario as an independent "event" in probability theory, and evaluate the partial preferable probability of the utility value of each benefit and the total preferable probability of each scheme according to PMOO;

c) The independency among "events" can be checked and verified by linear regression analysis. If the linear correlative factor between two events is less than the critical value r_c of linear correlative factor, the two events could be designated as independent, otherwise they are designated as dependent. The critical value r_c of linear correlative factor can be found from common statistical book [5]. For the two dependent events, only one event remains to join the evaluation. In case of the linear correlative factor with negative value, the corresponding both events join the evaluation;

d) Find the one with highest total preferable probability to be as the proper scheme (solution) of the problem in viewpoint of systems theory [4]. According to systems theory, the optimization of a system is its overall optimization [4].

The rationality of this operation for decision-making lies in bringing all kinds of scenarios into a system from the perspective of systems theory [4], the selection for a scheme with the highest total preferable probability is to take into account the needs of all kinds of scenarios together comprehensively.

Besides, the utility value of profit itself has the characteristics of “the bigger the better” or “the smaller the better”, which can be classified as the beneficial type of attribute or cost type of attribute of PMOO method correspondingly.

The obvious advantage of this option is that it could give excellent result in comprehensively considering all scenarios together in view of systems theory simultaneously, the solution is relatively stable and robust with respect to the variations of scenario due to the total preferable probability including full information of all scenarios.

3.2. Essence of the Perspectives of Systems Theory and PMOO

In the last paragraph, it illuminates that PMOO approach could be employed to solve the problem of unidentified scenarios by setting each scenario as an independent "event" in probability theory in the virtual system in view of “higher dimensional space” [4]. Then an appropriate option can be obtained with PMOO for these unidentified scenarios in comprehensively considering all scenarios together at the same time.

PMOO was specially proposed to conduct optimal problem with multiple objectives in comprehensively considering all objectives together simultaneously, which avoids the main perplexity of traditional methods of multi-objective optimization in ignoring the simultaneity of multiple objectives in the optimization [5-14], Such as, in SAW [5], WASPAS [6], TOPSIS [7], VIKOR [8], AHP, MORA [9], COPRAS [10], PIV [11], PSI [12], and PSIE [13], etc. For example, in the SAW method, if the multiple objectives with functions of $g_1(x)$, $g_2(x)$, ..., $g_p(x)$ are involved, they will be "added" together with the weighting factor w_j, the selection of the weighting factor w_j is chosen subjectively, and the objectives (or attributes) have to be "normalized" when the dimensions of the objectives are different, again the selection of the normalized factors (denominators) involved is also “subjective” without any objectivity. In some methods, artificial factors such as virtual "ideal point" are also introduced artificially.

Besides, "addition" operation is union in set theory, and in probability theory, "addition" is the "sum" of events in principle. Therefore, the operation mode of "addition" essentially neither coincides with the original intention of simultaneity of optimization with multiple objectives nor reflects the original intention of unsubstitution of the multiple objectives. Pareto method can provide a set of solutions only.

This situation indicates that the previous methods concerning optimization with multiple objectives are unsatisfied. The Pareto approach could only provide the Pareto front which is set of solutions instead of an optimum solution [14].

Therefore, in order to reveal the original intention of simultaneity and unsubstitution of the multiple objectives in optimization, other routine must be explored.

From the perspective of probability theory, there is an index of “joint probability” that could reflect the simultaneous appearance of two independent events, it is the "product" of the probabilities of the two independent events, and it belongs to some type of "intersection" of the two sub-sets in set theory. On the other hand, the unsubstitution of two independent events is included as well.

In view of this case, a probabilistic multi-objective optimization (PMOO) was put forward recent year. It regards the problem of optimization of multiple objectives as a system from the point of view of systems theory. According to this standpoint, "the optimum position of multiple objectives" in a system is "the best position of the entire system as whole". Furthermore, by considering the operation approaches of probability theory and set theory in dealing with the problem of simultaneous appearance of two independent events and their unsubstitution, the a idea of "preferable probability" and its assessment are put forward, which is based on systems theory and probability theory. Thus the theory and methodology of PMOO was established [4].

Besides, the assessed attributes (objectives) of alternative schemes in the optimization are primitively classified into two types, i.e., beneficial type and cost type, and the correspondingly quantitative assessments of their partial preferable probabilities are formulated. The optimization problem with multiple objectives is considered as an overall optimization of a system, the simultaneity of optimization of multiple objectives is revealed by the joint probability problem of the appearance of multiple individual events in probability theory with an objective being analogically seen as a pseudo-event. Therefore, the total preferable probability of each alternative scheme is the product of partial preferable probabilities of all possible independent objectives of the scheme in the system. Finally, the one with the largest total preferable probability is chosen as the optimum result [4, 13].

In addition, the robust design of industrial processes and products can also be analyzed and discussed in framework of PMOO method systematically [4, 13].

The procedure of PMOO evaluation is shown here in Fig. (**1**).

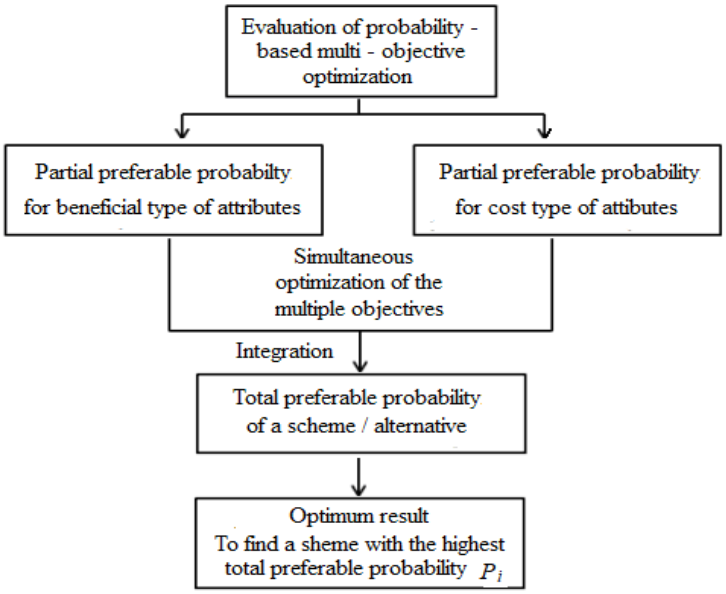

Figure 1: Evaluation procedure of PMOO approach.

For the beneficial type of attribute, its partial probability is evaluated by using Eq.(1),

$$P_{ij} = \alpha_j X_{ij},\ \alpha_j = 1/(n \cdot \overline{X_j}),\ \text{i = 1, 2, ...n; j = 1, 2, ...m.} \tag{1}$$

While for cost type of attribute, its partial probability is evaluated by using Eq.(2),

$$P_{ij} = \beta_j (X_{j\max} + X_{jmin} - X_{ij}),\ \beta_j = 1/[n \cdot (X_{j\max} + X_{jmin} - \overline{X_j})]. \tag{2}$$

The total preferable probability of a scheme / alternative is evaluated by using Eq. (3),

$$P_i = P_{i1} \cdot P_{i2} \cdots P_{im} = \prod_{j=1}^{m} P_{ij}. \tag{3}$$

In Eq. (1) through Eq. (3), P_{ij} indicates the partial preferable probability of the *j*-th scene utility index of the *i*-th alternative scheme X_{ij}; *n* is the total number of alternative schemes; *m* reflects the total number of scenes; $\overline{X_j}$represents the arithmetic average utility value of the *j*-th scene within the involved schemes; X_{jmax} and X_{jmin} are the maximum and minimum utility values of the *j*-th scene, respectively; α_j indicates the normalization factor of utility index of the *j*-th scene X_{ij} of beneficial type of attribute, and β_j is the normalization factor of utility index of the *j*-th scene X_{ij} of the cost type of attribute, P_i is the total preferable probability of the *i*-th scheme.

3.3. Application Examples

In the following, examples are presented to show the procedure in detail.

Example 1. Again the example in section 2 is employed conveniently. Table **6** shows the corresponding information [1], which involve 5 schemes (K_1, K_2,..., K_5) and four states (τ_1, τ_2,..., τ_4).

Linear regression analysis indicates that the natural states τ_1 and τ_2 are dependent with a linear regression factor of 0.9208, which is bigger than the critical value r_c of linear correlative factor r_c = 0.878 at α = 5% level [15] statistically, thus only the natural states τ_1 is remained in the evaluation. Of course, if τ_1 is remained in the evaluation, there might be little different, but the errors is in α = 5% level statistically.

The evaluated results by means of PMOO are given in Table **6** as well, i.e., the results of partial preferable probability, total preferable probability, and ranking. As a result, the scheme K_1 is with the highest value of total preferable probability and in rank 1, therefore it is the optimum option by means of PMOO.

Obviously, as compared to those options given in the section 2, here it results in scheme K_1 as the optimum scheme by means of PMOO, the distinguish characteristic of scheme K_1 is that it is with relatively higher average value of utility and stability with respect to variations of scenes, i.e., the scheme K_1 is robust to all scenes from τ_1 to τ_4 comprehensively, which reveals the compatibility of the scheme with all scenes. In addition, the physical meaning of this result is clear. Fortunately, the equal likelihood criterion plus stability assessment for schemes and pessimism criterion also give the same consequence coincidentally, however neither the equal likelihood criterion nor pessimism criterion has the essential of robustness and compatibility.

Table 6: A decision profit table for PMOO evaluation.

Profit / Scheme	State				Partial Preferable Probability			Total Preferable Probability, Rank	
	τ_1	τ_2	τ_3	τ_4	P_1	P_3	P_4	$P_i \times 10^2$	Rank
K_1	4	5	6	7	0.2353	0.2308	0.2059	1.1179	1
K_2	2	4	6	9	0.1176	0.2308	0.2647	0.7187	3
K_3	5	7	3	5	0.2941	0.1154	0.1471	0.4991	4
K_4	3	5	6	8	0.1765	0.2308	0.2353	0.9582	2
K_5	3	5	5	5	0.1765	0.1923	0.1471	0.4991	4

Example 2. The profit table of a problem is shown in Table **7** for a decision – making of investment [16]. It is with utility values of profit value, which involve 3 schemes (S_1, S_2, S_3) and three cases (C_1, C_2, C_3) [6]. The evaluated results in terms of optimism criterion (O), pessimism criterion (P), compromised criterion (C), equal likelihood criterion (E), and minimax regret criterion (R), etc. are shown as well. From Table **7**, it shows that there exist significant differences among different criteria.

Linear regression analysis indicates that there is no linear correlation among cases C_1, C_2 and C_3 at α = 5% level statistically.

The evaluated results by means of PMOO are given in Table **8** according to Fig. (**1**) and Eq. (1) through Eq. (3). As a result, the scheme S_2 is with the highest value of total preferable probability and in rank 1, it is the optimum option by means of PMOO. The reason of the optimum option of scheme S_2 is that it is with relatively higher average value of utility and stability comparatively, which reveals the more compatibility of the scheme S_2 with all cases.

Table 7: A decision profit table for an investment.

Profit / Scheme	Case			Option Result in Terms of Previous Method*				
	C_1	C_2	C_3	O	P	C	E	R
S_1	800	550	300	800	300	600	550	200#
S_2	650	600	500	650	500#	590	583.33#	350
S_3	1000	400	250	1000#	250	700#	550	250

*Notice: *O*, *P*, *C*, *E* and *R* in Table **7** express optimism criterion, pessimism criterion, compromise criterion, equal likelihood criterion, and minimax regret criterion, respectively; the symbol # indicates the selected scheme of the corresponding criterion.

Table 8: Evaluated results by means of PMOO for an investment problem.

Profit / Scheme	Case			Partial Preferable Probability			Total Evaluation	
	C_1	C_2	C_3	P_{C1}	P_{C2}	P_{C3}	$P_i \times 10^2$	Rank
S_1	800	550	300	0.3265	0.3548	0.2857	3.3104	2
S_2	650	600	500	0.2653	0.3871	0.4762	4.8904	1
S_3	1000	400	250	0.4082	0.2581	0.2381	2.5079	3

4. Risk - Based Decision - Making

Risk - based decision - making (decision making with probability) is a decision - making when the natural environment (state) is not completely determined, but the probability of its occurrence can be pushed or predicted.

In general, this kind of decision – making has the following conditions:

1) It has a clear target for decision maker to reach to;
2) It has more than two natural states which are independent of the will of the decision makers;
3) It has more than two schemes provided for option;
4) It has utilities of profit and loss value for each scheme in different natural states;
5) It has occurring probability of each natural state.

Till now the quantitative decision-making methods of risk - based decision - making form following approaches:

1) Maximum possibility criterion (MP). It selects the natural state with the greatest probability of occurrence, and selects the scheme with the greatest (best) benefit, regardless of other states;
2) Expected value criterion (EV). Firstly, it calculates the expected profit of each scheme according to the probability of each natural state, and then selects the scheme with the greatest (best) profit as the decision-making scheme;
3) Decision trees method (DT). Firstly, it draws a decision tree. Then it gives probability of each event, and calculates utilities of profit and loss value for each event. Finally, it determines the option according to the utilities of profit and loss value, the larger is selected.

It is obvious that the decision-making scheme selected by the above methods can't give overall consideration to the effects or influences of various natural states distinctly. It needs an appropriate selection to meet the demands of this case.

Again, it is expected to use PMOO approach to solve this problem by taking each natural state as an independent "event" in probability theory in viewpoint of systems theory, and find out the partial preferable probability of the utility value of each benefit and the total preferable probability of each scheme, and then select the one with highest total preferable probability [4].

Since the occurring probability of each natural state is known, the PMOO with probability for each attribute can be used directly to get the total preferable probability of each scheme. In the following, an example is given to show the application procedure of PMOO approach in this case.

Example 3. A factory needs to make a production plan for the next year [1]. According to market research and forecast analysis, it is concluded that the market sales of its products can be divided into three natural states: good (C_1), medium (C_2) and poor (C_3), and the probability ω_j (j = 1, 2, and 3) of these three natural states is 0.3, 0.5 and 0.2 respectively. The production of this factory can be carried out according to three schemes: large batch S_1, medium batch S_2 and small batch S_3, and the utility values of the corresponding profit and loss values are shown in Table **9**. Now it is necessary to determine its reasonable production plan, so that enterprise can obtain greater benefits. The evaluated results from previous methods are given in Table **9** as well.

The evaluation results by means of PMOO are given in Table **10** according to Fig. (**1**) and Eq. (1) through Eq. (3). As a result, the scheme S_2 is with the highest value of total preferable probability and ranked in No. 1, it is the optimum option by means of PMOO. In the evaluation, the formula, $P_t = \prod_{j=1}^{3} P_j^{\omega_j}$, is employed, ω_j is the occurring probability of the natural state C_j (j =*1*, *2*, and *3*).

Table 9: Utility values of the profit and loss values for production plan.

Scheme \ Profit	Case			Evaluated Result from Previous Method		
	C_1, ω_1=0.3	C_2, ω_2=0.5	C_3, ω_3=0.2	MP	EV	DT
S_1	20	12	8		13.6	13.6
S_2	16	16	10	√	14.8√	14.8√
S_3	12	12	12		12	12

***Notice:** *MP*, *EV*, and *DT* in Table **9** express maximum probability criterion, expected value criterion, and decision trees method, respectively; √ indicates the selected scheme of the corresponding criterion.

Table 10: Evaluation results by means of PMOO for production plan.

Scheme \ Profit	Case			Partial Preferable Probability			Total	
	C_1	C_2	C_3	P_1	P_2	P_3	P_i	Rank
S_1	20	12	8	0.4167	0.3	0.2667	0.3234	2
S_2	16	16	10	0.3333	0.4	0.3333	0.3651	1
S_3	12	12	12	0.2500	0.3	0.4000	0.3009	3

5. On Estimation of Utility in Risk - Based Decision – Making Condition

In the previous sections, the profit data is used as "utility value" directly since all profit data are all in positive form. In other cases, profit data might be in negative form, i.e., loss value. For the latter case, the estimation of "utility value" of profit and loss value is particular [1-3]. Bernoulli once proposed special approach to estimate "utility value" of profit and loss value with negative data to reflect the preference degree relatively [1-3], the procedure is rather complicated. Here in this section, we recommend a simply linear approach to relatively reflect the preference degree of data in each natural state, which takes a neutral attitude towards risks, i.e., setting the utility value of the most willing and most favorite things to 100, and the utility value of the least willing and least favorite things to 0. The utility value of others in the intermediate state is also in between, and can be estimated linearly. In the following, an example is given to show the application procedure of PMOO approach in this case.

Example 4. There is a decision – making thing for production plan to be conducted [1], its profit and loss value is listed in Table **11**. Obviously, in this example the expected value of each scheme is nearly the same, i.e., 275, it is difficult to employ expected value criterion to solve this decision – making problem. The utility value of this problem by using current linear estimation is shown in Table **12**. After completing the estimation of utility value, the decision – making can be conducted by means of PMOO subsequently, Table **13** shows the evaluation result in terms of PMOO approach. The evaluation is conducted by using following formula, $P_t = \prod_{j=1}^{4} P_j^{\omega_j}$.

From Table **13**, it derives scheme S_1 as the optimal option. Obviously, S_1 is the appropriate option relatively in this problem, it is comparatively stable, see Table **13**.

Table 11: A decision – making thing for production plan.

Scheme \ Profit	Case				Expected Value
	C_1, ω_1=0.35	C_2, ω_2=0.35	C_3, ω_3=0.15	C_4, ω_4=0.15	$E(S_i)$
S_1	418.5	418.5	-60	-60	274.95
S_2	650	-100	650	-100	275.00
S_3	483	211.3	480	-267	274.96

Table 12: Utility value in terms of simply linear approach.

Utility / Scheme	Case			
	C_1, ω_1=0.35	C_2, ω_2=0.35	C_3, ω_3=0.15	C_4, ω_4=0.15
S_1	74.75	74.75	22.57	22.57
S_2	100.00	18.21	100.00	18.21
S_3	81.79	52.16	81.46	0

Table 13: Evaluation result in terms of PMOO approach.

Scheme	Partial Preferable Probability				Total Evaluation	
	P_1	P_2	P_3	P_4	P_i×10	Rank
S_1	0.2914	0.5151	0.1106	0.5535	3.3867	1
S_2	0.3898	0.1255	0.4901	0.4465	2.7690	2
S_3	0.3188	0.3594	0.3993	0	0	3

In fact, utility value is a word with special meaning, which can reflect people's psychological satisfaction, endurance and values on different occasions. The same thing, for example one US dollar ($), will give people different satisfaction on different occasions. When you plan to buy a commodity with psychological evaluation or forecast in 1000 US dollar ($), but you encounter a commodity with a price of 1008 US dollar ($), you will feel suitable, feel comfortable psychologically and think about value for money; On the other hand, when you plan to buy a commodity with psychological evaluation or forecast in 1 US dollar ($), but you encounter a commodity with a price tag of 2 US dollar ($), you feel inappropriate, even psychologically unacceptable, and wonder how it can be so expensive. Besides, utility value is a relative value, which is an index used to quantify (measure) the attitude towards risk, different person could have different attitude towards risk. Therefore, the estimation for utility value might have different approaches specially [17-21]. After quantification to his attitude, the evaluation in terms of PMOO can be performed.

6. Discussions

Durbach *et al*. pointed out that the formal modeling of risks and uncertainties in multi-criteria decision analysis (MCDA) involves treating uncertainties both in *internal uncertainty* and *external uncertainty* [22]. The *external uncertainty* is the uncertainty for scenarios. They also stressed the robustness to scenarios and antifragility of a solution, a clearly robust solution would be one which has maximum aggregate value (or nearly so) under all scenarios [22-25]. Applications have focused on assessing the stability of performances over scenarios by directly presenting the results with information about the range of performances across scenarios [23]. In a current work, Durbach *et al.* have demonstrated that in some cases at least the arguments of absolute performance – augmented by an aggregation with maximizes the worst performance across scenarios – or relatively small regret compared to the alternatives across a wide range of plausible futures may be misleading [23]. The approach is naturally extended to the more analytical process of designing, evaluating and selecting courses of action on the basis of robustness to these uncertainties [24]. The approach is, however, naturally extended to the more analytical process of designing, evaluating and selecting courses of action on the basis of robustness to these uncertainties, which suggests close parallels with MCDA [24].

It is clear that above statements stressed the compatibility of an effective approach to all scenarios.

As to robustness designs, it is involved in many engineering problems, such as climate change research [26], mechanical design [27], design of experiment [28], industrial complex structures from local to global non-intrusive

coupling strategy [29], load fluctuations in engineering [30], etc. Lempert pointed out that seeking robust strategies provides an animating idea for robust decision making, which includes choosing a set of scenarios that include the most important uncertainties facing the users and then identifying strategies that perform well across all of them [26]. Hami *et al.* claimed that integrating uncertainty into the design process is a practice commonly used by engineers [27]. We may want to obtain a level of performance which is minimally sensitive to uncertainty [27]. When dealing with nonlinear regression models, the dependence of optimal experimental designs on reference parameter values, which may be uncertain, introduces uncertainty into the designs themselves. To address this, the objective function in the optimization of experimental design is modified to account for uncertainty. There are two predominant approaches, the first one is the average case approach, which replaces the objective function with its expected value concerning the uncertain parameter θ, and the second one is the worst case approach, which minimizes the worst case scenario, also known as the minimax approach [28]. Bubel *et al.* found that the average case approach is known to be too optimistic as it tends to ignore single bad scenarios [28]. This is particularly problematic, if the true scenario $\theta = \theta_t$ is one of those bad scenarios. In contrast, the worst case approach tends to be too pessimistic as it only focuses on the worst case scenario [28], generally leaving too much room for improvement in favorable scenarios. Industrial complex structures are generally represented by a system of connected models rather than a single unified model, with the need for communication between different scales to ensure consistency and accuracy in the simulation process. In this context, model analysis based on the level of maturity of the overall project is generally conducted in a sequential and partitioned manner [29]. Karaouni *et al.* proposed a complementary bottom-up approach ("agile design") that considers, from the preliminary design phases the potential impact of local changes on the overall architecture. This is the philosophy behind the concept of robust design, which seeks to enhance robustness to local variations and uncertainties [29]. The developments in smart grids have made it necessary to manage both sides to balance production and consumption [30]. Dogan *et al.* proposed a novel approach robust to voltage fluctuations [30]. The robust design and assessment of product and production is also conducted [4, 23].

From above discussion, it indicates the generality of robust design in various engineering fields, though different approaches act in corresponding area, its aim still is to maintain the stability of the solution.

PMOO approach could take full information of all objectives in multi-objective optimization into consideration simultaneously, so as to obtain a comprehensive solution in manner of joint probability. Furthermore, as to decision-making under uncertain scenario, a virtual system is constructed in view of "higher dimensional space", which includes full information of all the uncertain scenarios as independent events / attributes first. It enables us to propose a new quantitative approach for decision-making under uncertain scenarios in viewpoint of systems theory based on PMOO rationally. The solution of such virtual system could be with robustness and stable despite of the variations of scenarios, which is a comprehensive multi-scenario approach inevitably.

7. Concluding Remarks

In this article, a novel approach based on PMOO are put forward to provide quantitative decision-makings under uncertain scenarios in the constructed virtual system in view of "higher dimensional space" rationally, which is an overall approach comprehensively considering all scenarios together in coordinate way, and has the characteristic of relative stability or with robustness with respect to the variation of scenarios in some sense. The quantitative risk decision based on PMOO embodies its completeness and comprehensiveness.

Conflict of Interest

The authors declared that there is no conflict of interest involved.

Funding

This research received no specific grant from any funding agency in the public, commercial, or not-for-profit sectors.

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